UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MODULE	MAT01A1 CALCULUS OF ONE-VARIABLE FUNCTIONS
CAMPUS	APK
ASSESSMENT	EXAM

DATE 30/05/2017	TIME 12:30
ASSESSOR(S)	DR A CRAIG MS S RICHARDSON
INTERNAL MODERATOR	MR S MAFUNDA
DURATION 2 HOURS	MARKS 70

SURNAME AND INITIALS _____

STUDENT NUMBER _____

CONTACT NUMBER _____

NUMBER OF PAGES: 1 + 12 PAGES

INSTRUCTIONS:
1. ANSWER ALL THE QUESTIONS ON THE PAPER IN PEN.
2. NO CALCULATORS ARE ALLOWED.
3. SHOW ALL CALCULATIONS AND MOTIVATE ALL ANSWERS.
4. IF YOU REQUIRE EXTRA SPACE, CONTINUE ON THE ADJACENT BLANK PAGE AND INDICATE THIS CLEARLY.

Question 1 [8 marks]

For questions 1.1 – 1.8, choose **one** correct answer, and make a cross (X) in the correct block.

Question	**a**	**b**	**c**	**d**	**e**
1.1					
1.2					
1.3					
1.4					
1.5					
1.6					
1.7					
1.8					

1.1 Find the vertical asymptotes of the function $y = \dfrac{2x^2+1}{3x-2x^2}$. (1)

(a) $x = 0$ only

(b) $x = \dfrac{2}{3}$ only

(c) $x = 0$, $x = \dfrac{2}{3}$

(d) $x = \dfrac{3}{2}$, $x = 0$

(e) None of the above

1.2 Find the point(s) on the graph of $f(x) = x^3 - 2$ where the slope is 3. (1)

(a) $(\sqrt[3]{2}; 0)$

(b) $(1; 3)$ and $(-1; 3)$

(c) $(1; -1)$ and $(-1; -3)$

(d) $(1; 3)$

(e) None of the above

1.3 Suppose $f(3) = 0$, $f'(3) = 6$, $g(3) = 1$, $g'(3) = \dfrac{1}{3}$, and $h(x) = \dfrac{f(x)}{g(x)}$. Evaluate $h'(3)$. (1)

(a) 18

(b) 6

(c) -6

(d) -2

(e) None of the above.

1.4 Let $f(x) = \dfrac{x^2 - x - 12}{x - 4}$. How would you define $f(4)$ to make f continuous at $x = 4$? (1)

(a) $f(4) = 0$
(b) $f(4) = 3$
(c) $f(4) = 4$
(d) $f(4) = 7$
(e) None of the above

1.5 If $2x - 1 \leqslant f(x) \leqslant x^2$ for $0 < x < 3$, find $\lim\limits_{x \to 1} f(x)$. (1)

(a) 1
(b) -1
(c) 0
(d) 3
(e) None of the above

1.6 Find $f(x)$ if $f'(x) = \sin x$ and $f(\pi) = 2$. (1)

(a) $f(x) = \cos x + 1$
(b) $f(x) = -\cos x + 3$
(c) $f(x) = -\cos x + 1$
(d) $f(x) = \cos x + 3$
(e) None of the above

1.7 $\sum\limits_{n=1}^{20} (-1)^n$ is equal to: (1)

(a) 0
(b) 20
(c) -20
(d) 1
(e) None of the above

1.8 The solution of $2\ln(-x) = 1$ is: (1)

(a) There are no solutions
(b) $e^{-\frac{1}{2}}$
(c) $-e^{\frac{1}{2}}$
(d) $\dfrac{1}{2}$
(e) None of the above

Question 2 [3 marks]

Solve the inequality: $\dfrac{-3}{x^2+4x} < 1$. (3)

Question 3 [2 marks]

Prove the identity : $\tan x + \tan y = \dfrac{\sin(x+y)}{\cos x \cos y}$ (2)

Question 4 [5 marks]

(a) If $u(x) = \tan^2(x^2 + 2)$ find $f(x), g(x), h(x)$ and $j(x)$ such that $u(x) = (f \circ g \circ h \circ j)(x)$. (2)

(b) Given $f(x) = \ln(-3 + \ln(2x))$

(i) Find the domain of f. (1)

(ii) Find the inverse function f^{-1}. (2)

Question 5 [4 marks]

Calculate the following limits **without** using L'Hospital's rule:

(a) $\lim\limits_{t \to 2} \dfrac{t^2 - 4}{t^3 - 8}$ (2)

(b) $\lim\limits_{x\to\infty} \dfrac{\sqrt{x^2-9}}{2x-6}$ (2)

Question 6 [4 marks]

Let $f : \mathbb{R} \to \mathbb{R}$ be a function and $a \in \mathbb{R}$. Prove the following statement: (4)

If f is differentiable at a then f is continuous at a.

Question 7 [3 marks]

Use the definition of the derivative at a point to calculate $f'(6)$ if $f(x) = \sqrt{x-2}$. (3)

Question 8 [9 marks]

Find the following derivatives:

(a) $\dfrac{dy}{dx}$ if $y = \sqrt{4+e^{2x}} + \sec x$ (2)

(b) $\dfrac{dy}{dx}$ if $\cos 3y = \tan 2x$ (2)

(c) $D_x\left[\dfrac{x^3}{3^x}\right]$ (2)

(d) $\dfrac{dy}{dx}$ if $y = -\coth\dfrac{1}{x} + \ln\sqrt{\tanh 2x}$ (3)

Question 9 [3 marks]

Show that the function $f(x) = x - \cos x$ has a root in the interval $(0, \pi)$. (3)

Question 10 [3 marks]

Find the limit: $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left(1+\frac{2i}{n}\right)^2$ (3)

Question 11 [2 marks]

Evaluate: $\lim_{x\to\pi^+} \frac{\sin x}{\sqrt{x-\pi}}$. (2)

Question 12 [4 marks]

Prove that $\lim\limits_{\theta \to 0} \dfrac{\sin \theta}{\theta} = 1$. (4)

Question 13 [6 marks]

Evaluate the following integrals:

(a) $\int_0^1 \left(\frac{1}{1+x^2} + \frac{\pi}{4}\right) dx$ (2)

(b) $\int \left(5x^2 + \frac{1}{x} - \sin x\right) dx$ (2)

(c) $\int x \cos(4x^2)\, dx$ (2)

Question 14 [3 marks]

Complete the following table with the correct truth values: (3)

p	q	r	$p \wedge q$	$r \to q$	$\neg r$	$\neg p \vee q$	$q \to p$	$(\neg p \vee q) \leftrightarrow \neg r$
T			F		F			

Question 15 [4 marks]

(a) Write the statement "All real numbers are equal to two or strictly less than zero" as a first-order formula. (1)

(b) Is the statement in (a) true? Justify your answer. (1)

(c) Write the negation of the statement from (a) in natural language. (2)

Question 16 [3 marks]

Use proof by cases to show that $3n - n^2$ is even for all $n \in \mathbb{Z}$.

Question 17 [4 marks]

Prove that $11^n - 6$ is divisible by 5 for all $n \in \mathbb{Z}^+$.